MATHEMATICS

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MODULE 1
PROBLEM SOLVING

Distance Learning

Alberta EDUCATION

LEARNING FACILITATOR'S MANUAL

Mathematics 8

Module 1: Problem Solving

LEARNING FACILITATOR'S MANUAL

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Student's should not have access to these assignments or the final test until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

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Learning Facilitator's Manual
Module 1
Problem Solving
Alberta Distance Learning Centre
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CONTENTS

COURSE INTRODUCTION 1

Overview of the Learning Package 1

Overview of the Junior High Mathematics Program of Studies 4
- Rationale and Philosophy 4
- Goals 4
- Program Dimensions 5
- Suggested Time Allocations 5

Overview of Mathematics 8 6
- Modules 7
- Assignment Booklets 9
- Equipment 9
- Media 9
- Mandatory Components 11
- Optional Components 11

Using This Learning Package in the Classroom 12
- Conventional Classroom 12
- Open-Learning Classroom 12
- Learning Package 13
- Support 13
- Management 14
- The Roles of the Teacher and the Student in an Open-Learning Classroom 15
- Evaluation 15
- Observation 16
- Individual Conferences 16
- Self-Appraisal 16
- Informal Evaluation: Assignments 17
- Final Test 17

Introducing Students to the System 18
- Reading Level 18
- Sequencing 19

MODULE 1: PROBLEM SOLVING 21
- Module Introduction 21
- The Four-Stage Process of Problem Solving 25
- Identifying the Problem 29
- Reasonableness of Answers 33
- Using Objects and Sketches 37
- Making Lists and Tables 43
- Guessing-Checking-Revising 49
- Finding and Applying a Pattern 53
- Simplifying a Problem 57
- Working Backwards 61
- More Than One Way to Go 65
- Module Conclusion 71
- Assignment 72

Teachers

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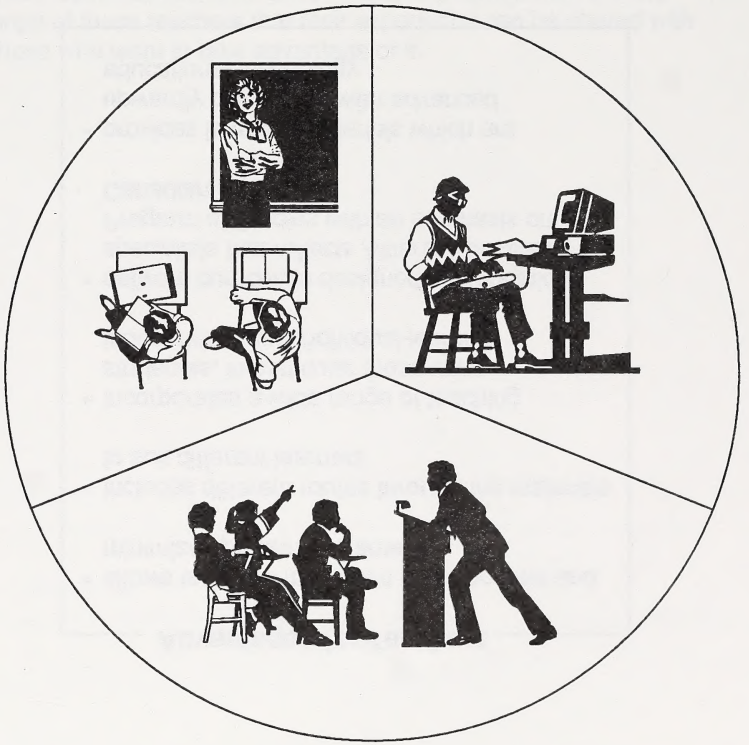
COURSE INTRODUCTION

Overview of the Learning Package

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

- ❒ Small-Schools Teacher
  - ❒ inexperienced
  - ❒ experienced, but in other subject areas
  - ❒ experienced in teaching mathematics, but wanting to try a different approach
- ❒ Distance-Learning Teacher
  - ❒ travelling to schools within the jurisdiction
  - ❒ using facsimile and teleconferencing to teach students within the area
- ❒ Larger-Schools Teacher
  - ❒ inexperienced
  - ❒ experienced in teaching mathematics, but wanting to try a different approach



Because this learning package has been created by experienced classroom teachers and distance learning specialists, it has many advantages for students and teachers regardless of their situation.

Advantages for Students

- incorporates a strong learner-centred philosophy
- promotes such qualities in the learner as autonomy, independence, and flexibility
- is developed through media which suits the needs and circumstances of the learner
- reflects the experiential background of Alberta students
- opens up opportunities by overcoming barriers that result from geographical location
- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time
- includes different routes through the materials to suit different learners
- incorporates a wide range of teaching strategies, in particular those using independent and individual learning
- delivers curriculum designed by education specialists that reflects Alberta Education Program of Studies with an emphasis on Canadian content
- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

The student materials are not the only components designed for independent, guided instruction; so is this Learning Facilitator's Manual. It begins with an overview of the current Alberta Education Program of Studies for junior high mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching junior high mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new courseware and determine how they might want to use it in their classroom.

Beyond the introduction the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The courseware and LFM are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.

Overview of the Junior High Mathematics Program of Studies

Rationale and Philosophy

This program was designed and developed using these assumptions:

- It is important to enhance a student's ability to solve problems.
- In learning new concepts students need to progress in stages from concrete, through pictorial, to symbolic.
- Students have varying learning styles and abilities which must be recognized.
- Calculators and computers are tools with which students must become familiar in order to function effectively in this technological age.
- Students need opportunities to practise new skills and to maintain previously developed skills.
- The progress students make in meeting mathematics objectives should be evaluated using both formal and informal methods.

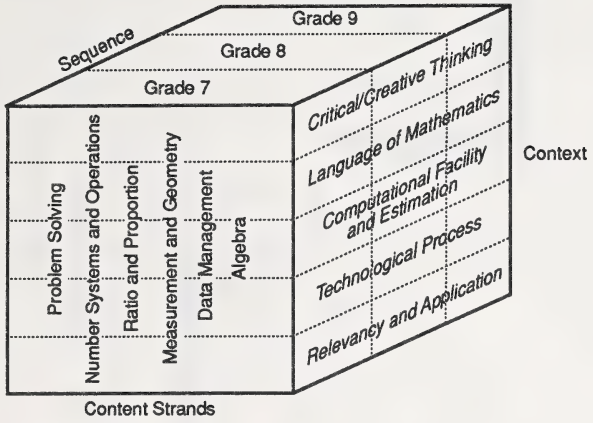
Goals

The goals of this program are to enable students to do the following:

- to grow in their capability to solve problems
- to use mathematics as a tool in the pursuit of personal goals and aspirations
- to develop good self-concepts and positive attitudes towards mathematics and lifelong learning

Program Dimensions

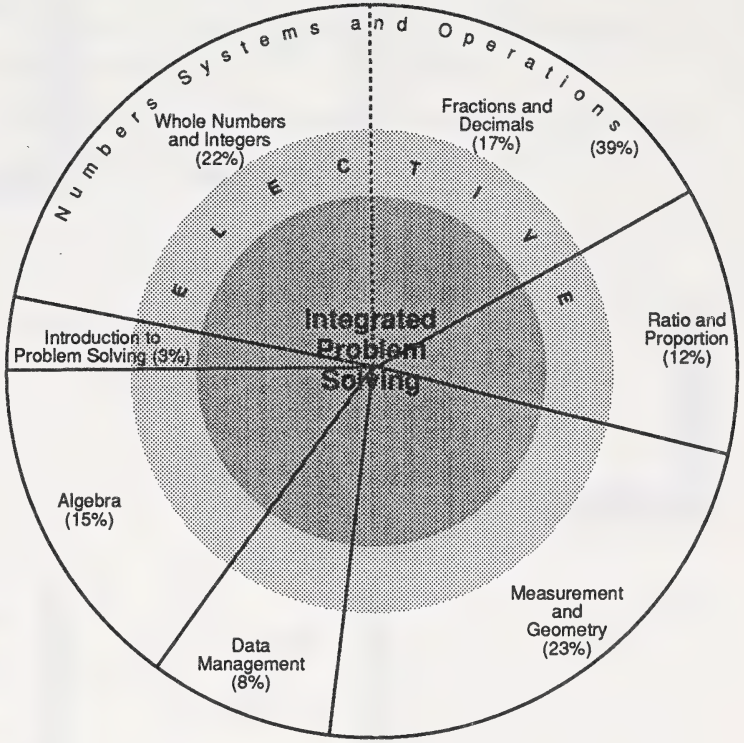
The dimensions of the Junior High Mathematics Program can be summarized by the following diagram:



The Junior High Mathematics Program has two components: the required component and the elective component. Eighty percent of the student's time shall be spent on the basic skills, knowledge, and attitudes outlined above. Twenty percent of the student's time shall be spent on activities meeting individual needs (remedial and/or enrichment). The enrichment shall be horizontal and not vertical; it is not intended to provide acceleration.

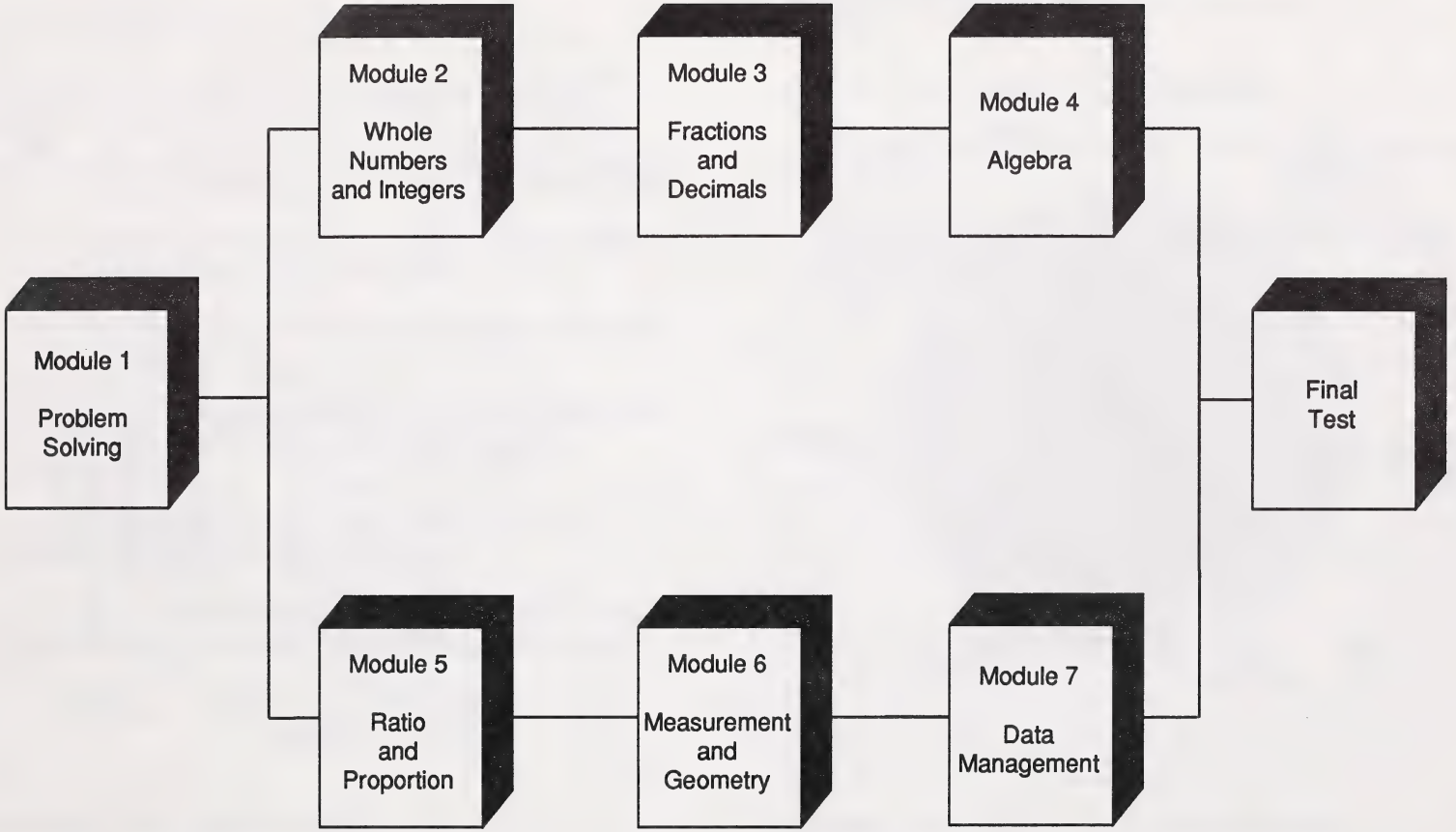
Suggested Time Allocations

The **minimum** requirement for the program is 100 hours. The breakdown of time is shown by the following diagram:



* Integrated Problem Solving - 20 % of time
** Integrated Elective (meeting individual needs) - 20 % of time

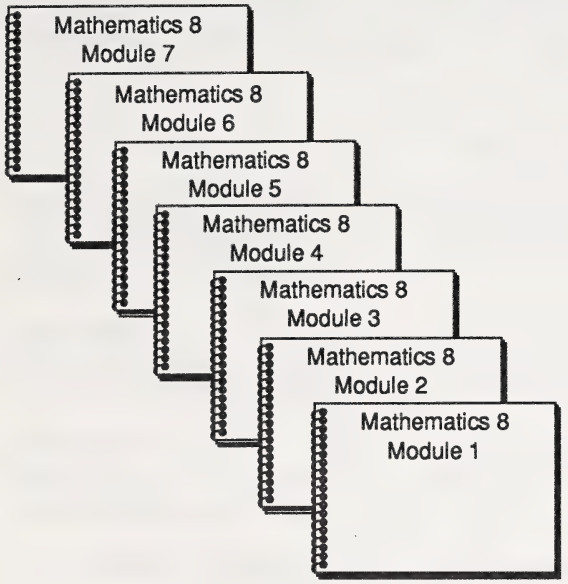
Overview of Mathematics 8



Mathematics 8 has seven modules and a final supervised test.

Modules

The print components include seven booklets called modules. These modules contain guided activities that instruct the students in a relevant, realistic setting.



The modules have many helpful features.

- Module 1 introduces problem solving, and this skill is emphasized throughout the package.
- Pretests are included to help you determine the student's strengths and weaknesses in each strand.
- Background material is included for students who have not mastered previously developed skills, so they can make a smooth transition.
- New skills are developed carefully and fully, and there is an abundance of practice.
- Games, manipulative activities, and math trivia are included to make mathematics fun and interesting.

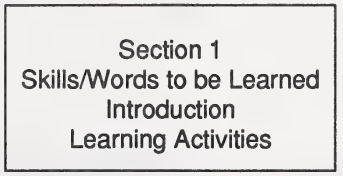
The modules have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated the teaching strategies of working from the concrete to the abstract, linking the old to the new, getting students actively involved, using advanced, intermediate, and post organizers, and many other techniques to enable learners to learn on their own for at least some of the time.

The modules in Mathematics 8 follow a systematic design.

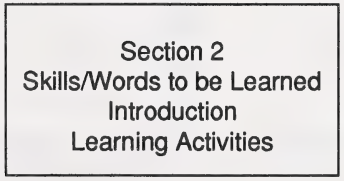
Each module booklet begins with a table of contents and a module introduction.

The module booklet is then broken into sections. The number of sections will vary from module to module, but each section has the same design:

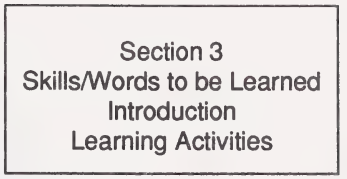
- The skills and words to be learned are listed under the heading "What Lies Ahead."



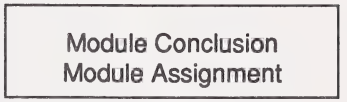
- There is an informal introduction to help motivate the student and explain the nature of the section.



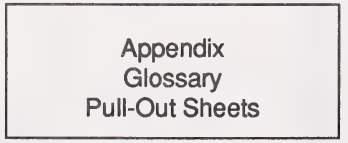
- There is a sequence of activities to learn and to practise the skills and words cited.



A module conclusion directs the student to the Module Assignment in the Assignment Booklet.



The module booklet usually ends with an appendix that contains a glossary and pull-out sheets.



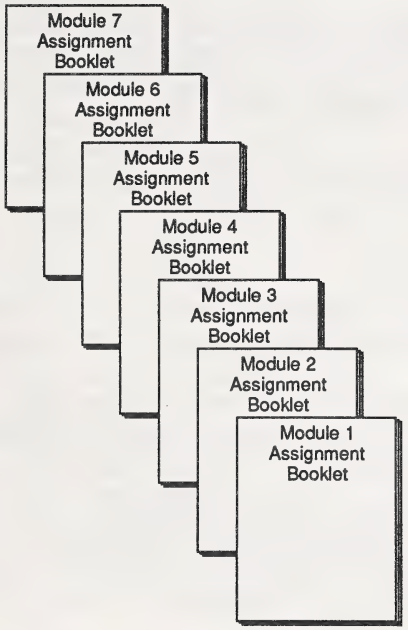
Note

The modules are very extensive in order to meet the needs of all students.

Students are **not** expected to do all the activities in the module booklets. You will help each student decide which activities are appropriate for his or her level of understanding and learning style preferences.

Assignment Booklets

Accompanying each module is an assignment booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete these assignment booklets when they have thoroughly reviewed the module materials. The assignment booklets have been designed for classroom use, for faxing, or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**


Module 7 Assignment Booklet
Module 6 Assignment Booklet
Module 5 Assignment Booklet
Module 4 Assignment Booklet
Module 3 Assignment Booklet
Module 2 Assignment Booklet
Module 1 Assignment Booklet

Equipment

Students require the use of a calculator and geometry set.

Media

Computer Disk Videocassette Course Audiocassette Providing General Teacher Guidance

The package also includes media options. Pathways have been developed so students can use computer disks and videocassettes to achieve the objective. These different routes have been included to suit different learners. Wherever videos or computer software have been included, a print pathway is also available. This way, if the media isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

A special audiocassette features a teacher guiding the student through the course. The appearance of the teacher icon reminds students that there is this additional help available. If the students are working individually, you may find this audiocassette a valuable asset. If the students are working in a large group, you may wish to guide the students yourself.

Video Programs

THINKABOUT: Find Your Guide (AIT)
THINKABOUT: There are Many Ways to Go (AIT)
MATH WORKS: Identifying the Problem (AIT)
MATH WORKS: Simplifying the Problem (AIT)
MATH WORKS: Adding and Subtracting Fractions and Mixed Numbers With Like Denominators (AIT)
MATH WORKS: Adding and Subtracting Fractions and Mixed Numbers With Unlike Denominators (AIT)
MATH WORKS: Using Mental Computation for Addition (AIT)
MATH WORKS: Using Mental Computation for Subtraction (AIT)
MATH WORKS: Sampling (AIT)
SOLVE IT: Guess-Check-Revise (AIT)
SOLVE IT: Solving a Simpler Problem (AIT)
SOLVE IT: Multiplication with Fractions and Mixed Numbers (AIT)
SOLVE IT: The Meaning of Percent (AIT)
SOLVE IT: Sampling (AIT)
MATHWAYS: Volume (kinetic)
MATH MOVES: Number Theory (ACCESS Network)
MATH MOVES: Adding and Subtracting Fractions (ACCESS Network)
MATH MOVES: Multiplying and Dividing Fractions (ACCESS Network)
MATH MOVES: Integers (ACCESS Network)
MATH MOVES: Equations—Solving With One Step (ACCESS Network)
MATH MOVES: Equations—Solving With More Than One Step (ACCESS Network)

Computer Programs

Problem-Solving Strategies (MECC)
Number Munchers (MECC)
Conquering Whole Numbers (MECC)
Conquering Fractions (MECC)
Math Strategies: Problem Solving (SRA)
Computer Drill and Instruction: Mathematics, Level D (SRA)
Integers Fast Facts (EduSoft)
Problem Solving in Algebra (Britannica)
Mathematics Activities Courseware 6, 7, 8 (Houghton Mifflin)
Geometric Concepts: Perimeter (Jostens Learning Systems)
Geometric Concepts: Area (Jostens Learning Systems)
LOGO

Mandatory Components

Equipment (Hardware)	Media	Materials
• audiocassette player • calculator • geometry set	• prepared audiocassettes (come with learning package)	• LFM for Mathematics 8 • One complete set of module booklets (7) and assignment booklets (7) for each student

Optional Components

Equipment (Hardware)	Media	Materials
• VCR • Computer	• videocassettes • computer programs • The Learning Management System™(LMS)	

Video programs used in this course may be available from your regional media centre, or from ACCESS Network.

Computer programs used in this course may be available from your regional media centre, the Learning Resources Distributing Centre, ACCESS Network, or a computer software supplier (see "computer supplies" in the Yellow Pages of your telephone book).

The Learning Management System™ (LMS) is available for this course. To access LMS you require a Digital Equipment Corporation (DEC) VAX™ computer system. Further information may be obtained from the

Alberta Distance Learning Centre
Box 4000
Barrhead, Alberta
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Telephone 674-5333

Using this Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows, or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open-Learning Classroom

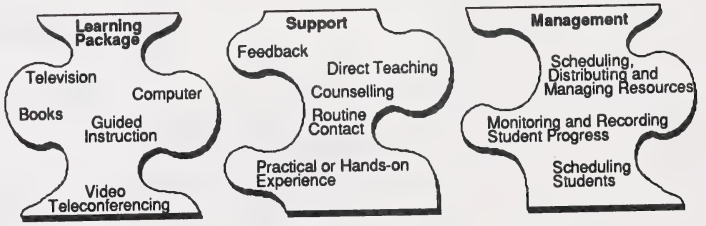
Open-learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed, due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centered approach is now possible. Improved technology now allows you to provide support to learners, individually, regardless of their pace or location. A teacher cannot be in 28 places at one time offering their guidance—media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own, for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

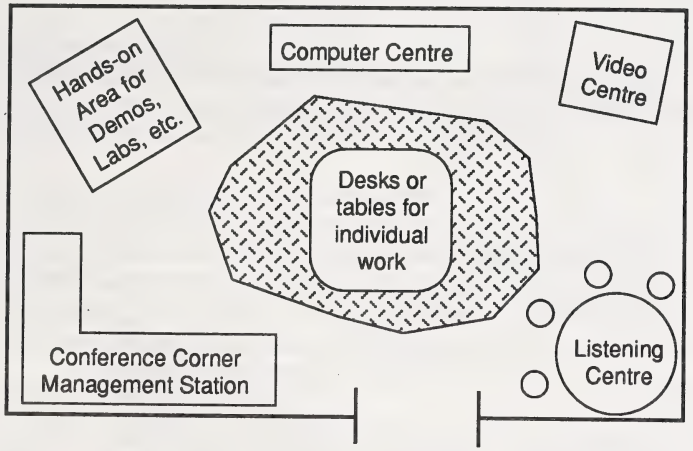
The Key to a Successful Open-Learning System



Learning Package

The specially-designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a centre can be established for specific media:



You may not have the luxury to have enough hardware to set up a permanent video or computer center in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every 3 - 5 days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students if necessary, or reserve media as required.

Support

Support is definitely a key element for any successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for each of the activities included in the module booklet.

The answers, explanations, and examples for each of the module activities are included in this LFM. You may decide to set up an answer station with colour-coded cards or you may want the students to come to you to discuss the activity together. As you and the students become more comfortable with an individualized system, you might increase the student's responsibilities—spot checking only to reinforce proper behaviour and to assess the student's day-to-day progress.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for doing these activities carefully before attempting the assignments in the Assignment Booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a bi-weekly conference scheduled by you, or as students reach a certain point (e.g. after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g. election, eclipse, etc.), or something prescheduled like the demonstration of a process, or involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open-learning system is essential to the succes of the program. The following areas need action to ensure that the system runs smoothly:

- Scheduling, Distributing and Managing Resources—As discussed earlier, this may require a need for centres or a system for students to project and reserve the necessary resources.
- Scheduling Students—Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may push to continue for long periods of time (e.g. all morning), teachers should discourage this. Concentration, retention, and motivation is improved by taking scheduled breaks.
- Monitoring Student Progress—You will need to record when modules are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.

Sample of a Student Progress Chart

Mathematics 8	Module 1	Module 2	Module 3	Module 4	Module 5	Module 6	Module 7	Final Mark
P Billy Adams A								
P Joe Bloom A								
P Sandy Nepoose A								
P = Project Completion Date A = Actual Completion Date								

The student could keep a personal log as well.

Such tracking of data could be stored easily on a computer.

- Recording Student Assessments—You will need to record the marks awarded to each student for work completed in each module Assignment Booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria is, it should be made clear to all students at the beginning.

Sample of a Student Assessment Chart

Math 8	Mod. 1	Mod. 2	Mod. 3	Mod. 4	Mod. 5	Mod. 6	Mod. 7	Final Test	Final Mark
Billy Adams	67	65	54	47	78	67	63	63	63
Joe Bloom	43	50	54	55	48	42	49	52	50
Sandy Nepoose	65	65	66	68	67	70	67	65	66

Letter grading could easily by substituted.

- Recording Effectiveness of System—Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Module 1			
Date	Module Booklet	Assignment Booklet	Resources/Media

The Roles of the Teacher and the Student in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on the continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centered not teacher-centered. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data gathering, processing, and decision making, both at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially-designed learning packages contain many kinds of informal and formal evaluation: observation, individualized conferences, self-appraisal, informal evaluation, assignments, and final tests.

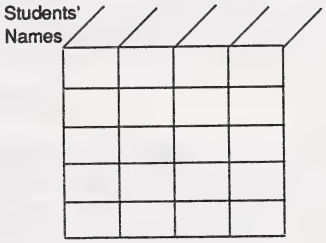
Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

- Does the student approach the work in a positive manner?
- Is the student struggling with the reading level?
- Does the student make good use of time?
- Does the student apply an appropriate study method?
- Can the student use references effectively?



Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar or at certain points in the module, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and they can assess the student's attitudes toward the subject, program, school, and self, as well as his or her relationship with other students. With guided questions, the teacher can encourage oral self-assessment: the student can discuss his or her own strengths or weaknesses in regard to the particular topic, module or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. By including activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process.

For variations, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the questions and responses to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for students. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices. If the student has access to a video player or a computer, media options are available. If not, print options must be chosen. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize your material to remember it?
- What steps do you follow when doing an assignment in your assignment booklet?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this module?

A chart or checklist could be used for recording responses.

Informal Evaluation: Assignments

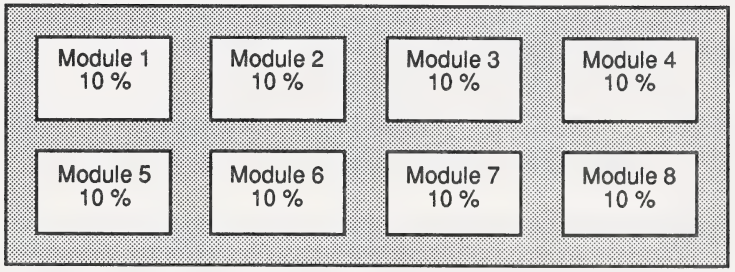
Informal evaluation such as the assignments included in each module, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behavior and attitudes that affect that achievement.

Each module contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the module. **The Student's mark for the module may be based solely on the outcome of learning evident in the Assignment Booklet; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning which outcomes will be evaluated, and that all students clearly understand what is expected.

Final Test

Each LFM includes a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the module booklets, gives the teacher precise information concerning what each student can or cannot do. Answers, explanations, and marking guides are also included.

The value of the final test and each module is the decision of the classroom teacher. Following is a suggestion only.



Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include what the advantages and the disadvantages might be of learning independently or in small groups. The roles of the students and teacher should be analyzed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

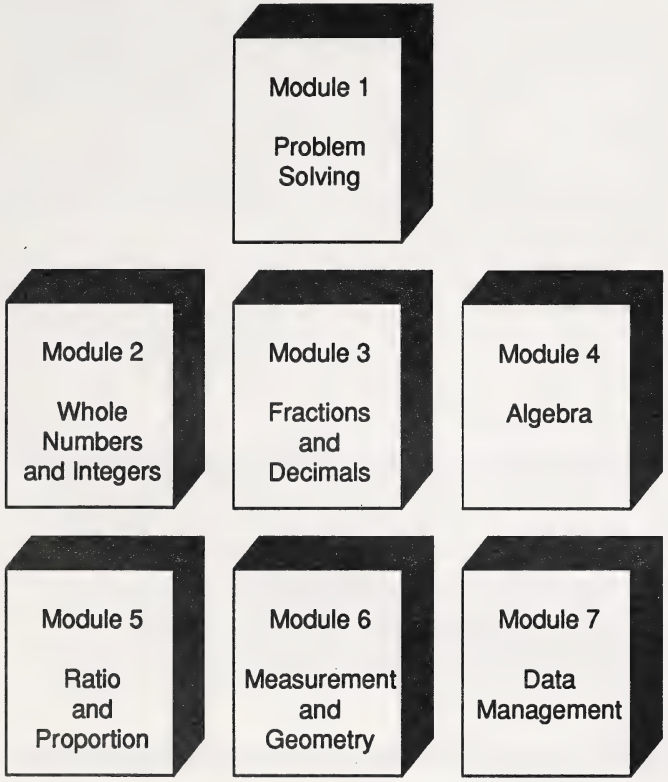
Reading Level

These course materials are largely print-based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks
- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post-organizers) to help give students a structure for incorporating new concepts.
- the recognition that vocabulary and concepts are basic to understanding content materials, and thus must be handled systematically (defined in context and in a specialized glossary)
- the acknowledgement that background knowledge and experience plays a vital role in comprehension
- the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software also as an alternative to print-based learning
- a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format
- the inclusion of media pathways and activity choices to encourage an active rather than passive approach
- instruction in a meaningful setting rather than in a contrived, workbook style
- using purposeful reading, viewing, and doing to produce better interpretation of the course materials
- the recognition that students need structured experiences when reading, viewing, or listening to instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary and extending. This structure closely resembles the reading process.

Sequencing

It is recommended that you start with Module 1 because this module includes basic introductory information. It is further recommended that you complete the modules in order because each module requires skills introduced in previous modules.



These materials are very comprehensive. It is important that you assure the students that the learning package is designed to build on what the student already knows. Students are not expected to do all the activities. Explain that you will help each student decide which activities are appropriate to his or her level of understanding and learning style preferences.

There are built-in pretests in Modules 2-7. These will help you and the student to recognize his or her strengths and weaknesses and to individualize the program. If the student already has a good understanding of the concepts to be taught or reviewed in a section, most of the section can be omitted. (We encourage the student to skim the section and do the concluding activity, however.) If the student's knowledge of the concept to be taught or reviewed is weak, all the necessary information is provided. (The student is encouraged to work through the entire section carefully.) There are also opportunities for the students to get additional reinforcement of difficult concepts if this is required.

These materials attempt to make mathematics real and fun. Every attempt has been made to give students a variety of meaningful activities and to give the students a chance to know when they have achieved mastery.

These materials also have integrated videos, computer software, and learning aids to add variety and cater to learning style preferences. Therefore, this learning package is more than just workbooks; it is a complete program.

MODULE INTRODUCTION

What Lies Ahead

In this section the student will preview Module 1 and learn how it will be evaluated.

Gathering Materials

For this section the student will need these items.

Mathematics 8
Module 1
Problem Solving

Guiding the Student

- You may wish to have the students read the Welcome and encourage the students to listen to the companion audiocassette for guidance, or you may wish to guide the students yourself.
- Discuss the learning process, time management, and evaluation with the students. Suggestions are given on the next page.

The Learning Process

Each section of Module 1 deals with a different aspect of problem solving. Students will learn about this approach by reading notes in the module booklet or by viewing a video program. Afterwards the student will be given different problems to practice. Sometimes there is a computer alternative for students who have Apple or other compatible computers. When the student completes a practice activity, you will help him or her check the answers and correct any errors. Emphasize to the student that the thinking process is more important than the answer. Encourage the student to explain how the answer was discovered.

Time Management

Decide how long the student will need to complete the module. An average student should spend about 2 weeks of a 40-week year to complete the module. It is recommended that students spend no more than 1 hour at a time doing mathematics.

Evaluation

Explain to the students how the module will be evaluated.

Additional Suggestions

The authors of this learning package recommend that you create a positive classroom atmosphere that allows students to express their own ideas and use individual approaches in problem solving. In order to develop this atmosphere you should do the following.

- Be supportive and encourage risk taking in finding solutions.
- Encourage students to use creative approaches.
- Be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate).
- Challenge students to think critically and justify strategies and solutions.
- Be enthusiastic and capable of recognizing the students' willingness and perseverence to solve problems.

The authors of this learning package further recommend that group work be used in problem solving. "A student in a group deals with ideas and questions from other members of the group and this may help each student progress in developing problem-solving strategies."[1]

The authors of this courseware also encourage the use of calculators in problem solving. "...time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used."[2]

In order to help students improve their problem-solving skills, it is also recommended that you observe and question students while they are solving problems.

Below are some of the factors you should consider:

- willingness to attempt problems
- use of a systematic approach
- selection of appropriate strategies
- efficiency in selection of appropriate strategies
- logical justification of strategies and solutions
- perseverence
- growth of confidence in problem-solving ability
- transfer of problem-solving skills to situations other than mathematics

Note

Evaluation techniques and instruments for problem solving are suggested in *Problem Solving Challenge for Mathematics* (Alberta Education, 1985) pp. 7, 8, 52-56.

After your students have been introduced to problem solving in Module 1, they will be given further problem-solving opportunities in other modules.

You may also wish to supplement the problems in the learning package with those from other books. (For example, many teachers like to have a problem of the week.)

[1–2]Alberta Education for the excerpts from the *Teachers Resource Manual for Junior High Mathematics.*

THE FOUR-STAGE PROCESS OF PROBLEM SOLVING

What Lies Ahead

In this section the student will learn the four stages of problem solving.

- understanding the problem
- developing a plan
- trying the plan
- looking back

Gathering Materials

For this section the student will need these items.

Mathematics 8
Module 1
Problem Solving

(optional)

THINKABOUT: Find Your Guide
(Agency for Instructional Television)

Guiding the Student

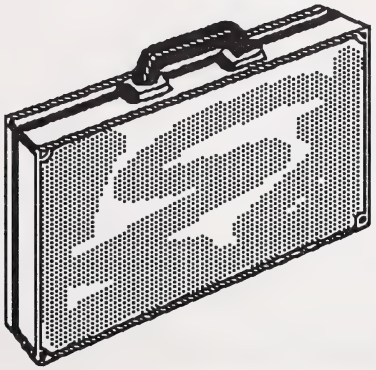
- Emphasize to the students the goal of the section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

1. Name the four stages in the process of problem solving.

2. Explain how you would solve each of these problems.

   a. If a briefcase was packed with $10 bills, what would be the total value of the money?

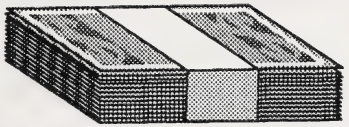


Suggested Answers

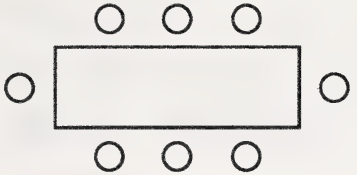
1. The four stages of problem solving are
   - understanding the problem
   - developing a plan
   - trying the plan
   - looking back

2. a. Answers will vary. A possible method is to use a briefcase and stacks of paper to represent stacks of bills. The students can then estimate the number of bills. The value of the money can be discovered by multiplying the number of bills by 10.

b. How would you cash a $33 cheque with six bills if you cannot use $1 bills?

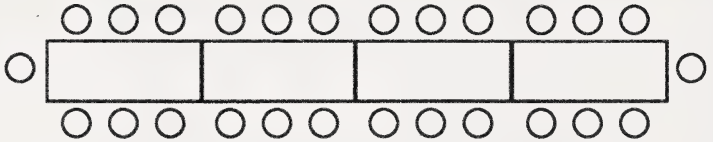


c. A table in a restaurant seats eight people—three at each long side and one person at each end. How many people can be seated if four of these tables are pushed together into one long row?



b. Through trial and error the student will discover that the cheque can be cashed using four $2 bills, one $5 bill, and one $20 bill.

c. Through using models or a diagram the student will discover the seating arrangement will look like this:



26 people can be seated if four tables are pushed together.

3. Jackie had the following problem to solve.

   Carlos earns $12.40 each week for his work stocking grocery shelves. He also receives an allowance of $4.25 each week for helping with chores around the house. How much money does Carlos make in a year?

   Jackie calculated that Carlos earns $8500 in a year. Is her answer reasonable? Why or why not?

4. Frank had this problem to solve.

   What digit goes in the ■, ◆, and ● to make the following equation true?

   ■■ × ●● = ■◆■

   Frank solved the problem this way.

   $33 \times 11 = 363$

   Are there any other possible answers to the problem? If so, what are they?

3. Carlos earns $12.40 + 4.25 or $16.65.
   There are 52 weeks in a year.

   This is an estimate of the amount earned in a year:

   $52 \times \$16.65 \doteq 50 \times \$20$
   $\doteq \$1000$

   $\$8500 > \$1000$

   So, Jackie's calculations are not reasonable.

4. Other possible answers are given:

   $22 \times 11 = 242$
   $44 \times 11 = 484$

IDENTIFYING THE PROBLEM

What Lies Ahead

In this section the student will learn these skills.

- identifying the essential elements of a problem
- ignoring unnecessary details
- changing the setting of a problem to help understand the problem
- restating the problem in their own words

Gathering Materials

For this section the student will need these items.

Mathematics 8
Module 1
Problem Solving

MATH WORKS: Identifying the Problem (Agency for Instructional Television)

Guiding the Student

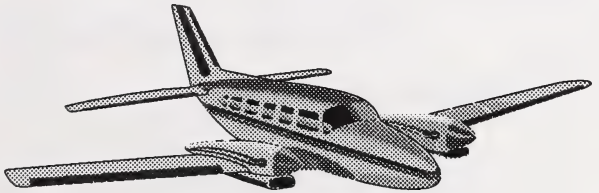
- Encourage the student to recognize the goal of the section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

1. Cross out the unnecessary details in the following problems and then restate the problems in your own words.

   a. Rock is working ~~very hard~~ to earn enough money to buy a ~~new~~ $425 bicycle. He gets paid $14 per week for babysitting ~~two hours on Thursday and five hours on Saturday~~. As well, he earns $20 per week for delivering newspapers ~~between 6:30 and 7:30 a.m. every morning~~. He also gets paid $5 ~~every Wednesday~~ for ~~spending $1\frac{1}{2}$~~ hours delivering ~~weekly advertising~~ flyers. For how many weeks will Rock have to work so that he can buy his new bicycle?

   b. A plane ~~leaves Vancouver at 7:30 a.m. and~~ arrives in Toronto at 12:00 noon Vancouver time. ~~It flies a distance of 3400 km at a cruising speed of 950 km/h~~. When it is 6:00 a.m. in Vancouver, it is 9:00 a.m. in Toronto. What time is it in Toronto when the plane lands?



Suggested Answers

1. a. Rock wants to buy a bicycle that costs $425. He earns $14 per week for babysitting, $20 per week for delivering papers, and $5 per week for delivering flyers. For how many weeks will Rock have to work before he can buy his bike?

   b. A plane arrives in Toronto at 12:00 noon Vancouver time. When it is 6:00 a.m. in Vancouver, it is 9:00 a.m. in Toronto. What time is it in Toronto when the plane lands?

2. Change the setting of the following problems.

a. A numismatist bought a rare coin for $30, sold it for $40, bought it back for $60, and sold it again for $70. How much money did the numismatist make or lose in buying and selling this coin?

b. Armond spent twice as much on shantung as he spent on chambray. If he spent $30 on chambray, how much did he spend on shantung?

2. a. Lisa bought a sweater for $30 and then sold it to a friend for $40. She bought the sweater back from her friend for $60 and sold it to another friend for $70. How much money did Lisa make or lose while buying and selling the sweater?

b. Randy's shirt cost twice as much as Cleo's shirt. If Cleo's shirt cost $30, how much did Randy's shirt cost?

REASONABLENESS OF ANSWERS

What Lies Ahead

In this section the student will learn these skills.

- estimating an answer
- determining if an answer is reasonable

Gathering Materials

For this section the student will need these items.

Mathematics 8
Module 1
Problem Solving

(optional)

SOLVE IT: Reasonableness of Answers (Agency for Instructional Television)

Guiding the Student

- Emphasize to the students the goal of the section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

For Questions 1 to 3, estimate the answers and then tell which of the provided answers is the most reasonable and explain why.

1. If a person walks 1 km, how many steps are taken?

   a. 25
   b. 100
   c. 2000

2. A hamburger patty has a mass of about 100 g. How much ground beef is needed if 25 people eat 2 hamburgers each?

   a. 5 kg
   b. 50 kg
   c. 500 kg

3. If you use 2 mL of toothpaste each time you brush your teeth (3 times a day), about how many weeks will it take to use up a 100-mL tube of toothpaste?

   a. 2 weeks
   b. 6 weeks
   c. 12 weeks

Suggested Answers

1. If you assume that the person will take 2 steps to cover 1 m, then 2000 steps will be required to cover 1000 m, which is equivalent to 1 km.

   c is the answer.

2. If 25 people could consume 50 patties, and each patty weighs 100 g, the total consumed will be 5000 g, which is equal to 5 kg.

   a is the answer.

3. If you use 6 mL of toothpaste each day, you will use 42 mL of toothpaste each week. 100 mL of toothpaste should last about 2 weeks.

   a is the answer.

For Questions 4 to 6, calculate the answers and then tell which of the provided answers is the most reasonable and explain why.

4. The 29 students in the seventh-grade class at the Willow Creek School are going on a field trip. If four students fit in each car, how many cars are needed?

   a. 7
   b. 1
   c. 7.25
   d. 8

5. Charlie's jazz band rehearsed a total of 29 hours in the last 4 days before the concert. If they spaced their rehearsals equally over the 4 days, how many hours did they practice each day?

   a. 7
   b. 1
   c. 7.25
   d. 8

6. Annette has collected 29 cassette tapes. She wants to arrange them in a box which will hold 4 tapes in each row. How many tapes will she put in the last row?

   a. 7
   b. 1
   c. 7.25
   d. 8

4. **d** is the answer.

   29 ÷ 4 = 7 R1

   If 7 cars are used, one student will be left behind. Since all 29 students must go, 8 cars must be used.

5. The answer is **c**.

   29 ÷ 4 = 7.25

   An exact answer is needed here. Fractions of an hour are possible.

6. The answer is **b**.

   29 ÷ 7 = 4 R1

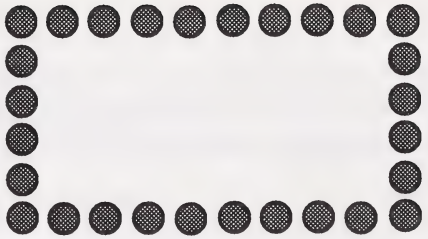
   Only 1 tape is in the last row since 28 tapes are required to complete 7 rows.

For Questions 7 and 8, draw a picture to help you get a reasonable answer.

7. Nadine is building a dog run. She wants the run to be a rectangle with 6 upright posts on each short side and 10 upright posts on each long side. How many posts will she need?

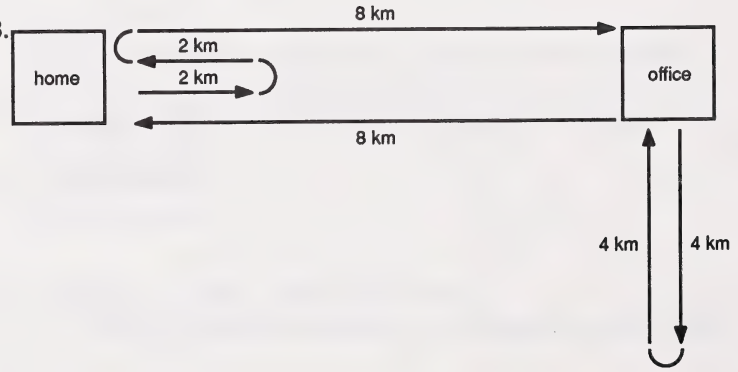
8. Gail's office is 8 km from her home. Yesterday morning she drove 2 km before she realized she had forgotten her briefcase. She returned home to get her briefcase and then drove to her office. At noon Gail drove 4 km to run some errands. She then went back to the office for the rest of the afternoon. At the end of the day she drove straight home. How far did she drive yesterday?

7.



Number of posts needed = 6 + 8 + 6 + 8 = 28
Nadine will need 28 posts.

8.



Number of km = 2 + 2 + 8 + 4 + 4 + 8 = 28
Gail drove 28 km.

USING SKETCHES AND OBJECTS

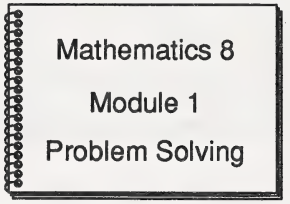
What Lies Ahead

In this section the student will learn these skills.

- using sketches to solve problems
- using objects to solve problems

Gathering Materials

For this section the student will need these items



MATH STRATEGIES: Solving Problems (SRA)

(optional)

Guiding the Student

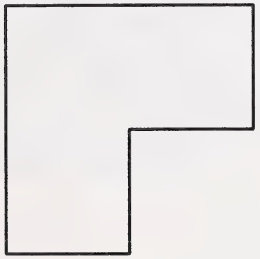
- Emphasize to the students the goal of the section.
- Help the students check the answers to the Practice Activities and correct any errors.

Practice Activities

1. Draw sketches to help you solve the following problems.

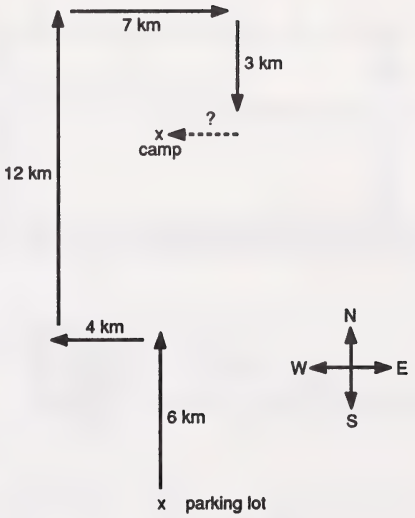
   a. A group of cross-country skiers intend to camp overnight at a site 15 km directly north of the parking lot. Starting at 8:00 a.m., they ski 6 km north, 4 km west, 12 km north, 7 km east, and 3 km south before stopping for lunch at noon. In what direction and how far must they ski after lunch to reach the campsite?

   b. A developer has bought this L-shaped piece of land. The developer wants to subdivide it into three congruent lots (lots that have the same shape and size). How can this be done?



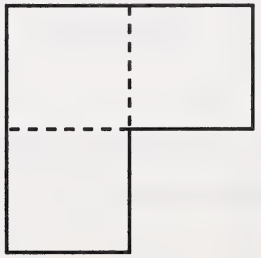
Suggested Answers

1. a.



They must ski 3 km west to reach the campsite.

b.



Three congruent lots can be made by subdividing as shown.

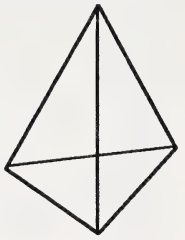
2. Use objects to help you solve the following problems.

   a. Assuming the length of a side of a triangle equals one toothpick, use six toothpicks to make four identical triangles.

   b. How would you fold this sheet of paper so that the numbered sections lie on top of one another in order from 1 to 8?

3	4	2	7
6	5	1	8

2. a. The solution is three dimensional. Only three of the toothpicks are on a flat surface.



   b. Step 1: Fold the sheet over like this so 6 lies on top of 5, and 3 lies on top of 4.

	2	7
	1	8

Step 2: Fold the sheet under like this. Now 6, 5, 4, and 3 are in order.

	1	8

Step 3: Fold the sheet under like this. Now 1, 2, 3, 4, 5, and 6 are in order.

1	8

Step 4: Fold the sheet under like this. Now all the numbers are in order.

1

Computer Alternative

3. Museum Trip

You'll need Disk 1 of *MATH STRATEGIES: Solving Problems* (SRA) to do this problem.

From the Chapter Menu choose "5 Using Models." Then choose "1 Museum Trip."

After you've read the problem, decide how much help you want.

Follow the directions on the screen.

Note: Whenever a smiling face appears, the computer stops. To make it continue press RETURN.

4. Measuring Puzzle

You'll need Disk 1 of *MATH STRATEGIES: Solving Problems* (SRA) to do this problem.

From the Chapter Menu choose "5 Using Models." Then choose "Measuring Puzzle."

Complete as many Measuring Puzzles as you wish at the computer.

3. Computer corrected

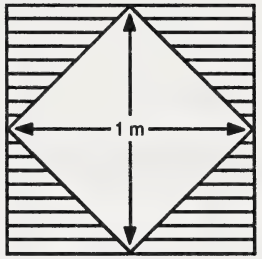
4. Computer corrected

Print Alternative

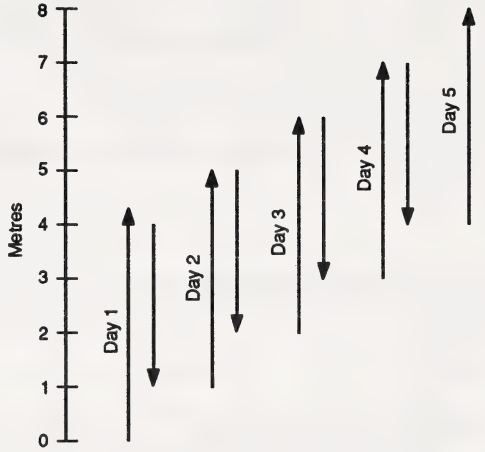
5. How can you cover half of a 1 m square window and still have a square opening that is 1 m across and 1 m from top to bottom?

6. A spider is at the bottom of a 8 m deep well. The spider climbs up 4 m each day, but slips back down 3 m each night. How many days will it take the spider to reach the top of the well?

5.



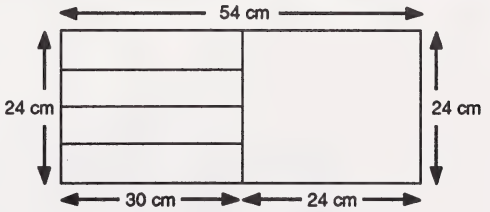
6.



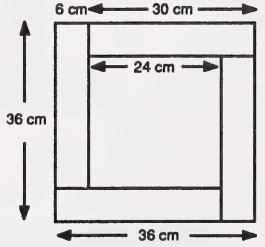
It will take 5 days to reach the top of the well.

7. Sandy wants to make a square breadboard from a rectangular piece of wood that measures 24 cm by 54 cm. She wants to use all the wood. What is the length of the side of the largest breadboard that Sandy can make?

7. The length of the largest breadboard that Sandy can make is 36 cm. It can be made by cutting the rectangular piece of wood this way



Assemble the breadboard this way:



The length of the largest breadboard Sandy can make is 36 cm.

MAKING LISTS AND TABLES

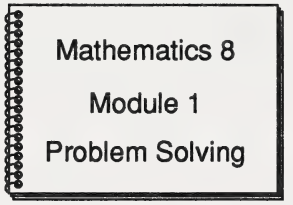
What Lies Ahead

In this section the student will learn these skills.

- making lists to solve problems
- making tables to solve problems

Gathering Materials

In this section the student will need these items.



Guiding the Student

- Emphasize to the students the goal of this section.
- Help the students check their answers to the Practice Acitivities and correct any errors.

Practice Activities

1. Kenza counted 29 wheels in a children's parade of bicycles and tricycles. If there were 12 children in the parade, how many tricycles were there?

2. Using only quarters, dimes, and nickels, how many ways can Philip pay for 95 cents worth of jelly beans?

Suggested Answers

1.

Number of Tricycles	1	2	3	4	5
Number of Bicycles	11	10	9	8	7
Number of Wheels	25	26	27	28	29

The table shows that there were 5 tricycles and 7 bicycles in the parade.

2.

Quarters	Dimes	Nickels
0	0	19
0	1	17
0	2	15
0	3	13
0	4	11
0	5	9
0	6	7
0	7	5
0	8	3

The chart is continued on the next page.

0	9	1
1	0	14
1	1	12
1	2	10
1	3	8
1	4	6
1	5	4
1	6	2
1	7	0
2	0	9
2	1	7
2	2	5
2	3	3
2	4	1
3	0	4
3	1	2
3	2	0

There are 26 ways that Philip can pay for 95 cents worth of jelly beans.

3. When choosing a new car at Magnificent Motors, the buyer may choose from 3 different models, 7 different colours, and either automatic or standard transmission. How many different car combinations are available?

3.

Model			Colour							Transmission	
1	2	3	1	2	3	4	5	6	7	Standard	Auto
✓			✓							✓	
✓			✓								✓
✓				✓						✓	
✓				✓							✓
✓					✓					✓	
✓					✓						✓
✓						✓				✓	
✓						✓					✓
✓							✓			✓	
✓							✓				✓
✓								✓		✓	
✓								✓			✓
✓									✓	✓	
✓									✓		✓

For Model 1 there are 14 different combinations. You can assume that Models 2 and 3 will also have 14 combinations each. Therefore, there should be 42 different car combinations in total.

4. Kimberley does housecleaning for her grandmother every Saturday. She earns 15 cents for each room that she vacuums and 25 cents for each room that she dusts. One day she did 16 jobs and was paid $3.30. How many rooms did she dust?

4.

Number of Rooms Vacuumed	1	2	3	4	5	6	7	8	9	10
Number of Rooms Dusted	15	14	13	12	11	10	9	8	7	6
Pay	3.90	3.80	3.70	3.60	3.50	3.40	3.30	3.20	3.10	3.00

Kimberley dusted 9 rooms.

GUESSING-CHECKING-REVISING

What Lies Ahead

In this section the student will learn these skills.

- solving problems by guessing, checking, and revising the answers
- organizing the guesses

Gathering Materials

For this section the student will need these items.

Mathematics 8
Module 1
Problem Solving

SOLVE IT: Guess-Check-Revise (AIT)

(optional)

Disk A of *MAC7* (Houghton-Mifflin)

(optional)

Guiding the Student

- Emphasize to the students the goal of this section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

Computer Alternative

1. Do the program "Guess and Test" on Disk A of *MAC 7* (Houghton-Mifflin).

Print Alternative

2. Carl opened his textbook and noticed that the product of the numbers of the two open pages was 2756. To which pages was the book opened?

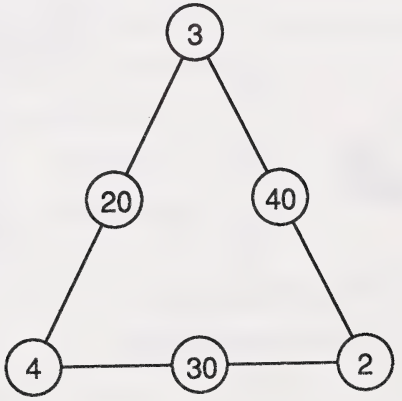
3. Place the numbers 2, 3, 4, 20, 30, and 40 in the circles so that the product of the three numbers on each side of the triangle is 240.

Suggested Answers

1. Computer corrected

2. The book was opened to pages 52 and 53.

3.



4. A physical education instructor spent $231 for 16 new balls. If softballs costs $4 each, footballs cost $24 each, and soccer balls cost $33 each, how many of each kind did the instructor buy?

Guess	Number of Softballs	Number of Footballs	Number of Soccer Balls	Test
1	3	4	9	3 × $ 4 = $ 12 4 × $24 = $ 96 9 × $33 = $297 Total 405
2	6	3	5	5 × $33 = $165 3 × $24 = $ 72 6 × $ 4 = $ 24 $261
3	3	4	4	4 × $33 = $132 4 × $24 = $ 96 3 × $ 4 = $ 12 $240
4	9	4	3	3 × $33 = $ 99 4 × $24 = $ 96 9 × $ 4 = $ 36 $231

The instructor bought 3 soccer balls, 4 footballs, and 9 softballs.

5. The sum of the present ages of two sisters is 24. Six years ago Chris was twice as old as Tina. What are their present ages?

Guess	Chris' Age Six Years Ago	Tina's Age Six Years Ago	Test
1	6	3	6 + 6 = 12 3 + 6 = 9 21
2	10	5	10 + 6 = 16 5 + 6 = 11 27
3	12	6	12 + 6 = 18 6 + 6 = 12 30
4	8	4	8 + 6 = 14 4 + 6 = 10 24

Chris is 14 and Tina is 10.

FINDING AND APPLYING A PATTERN

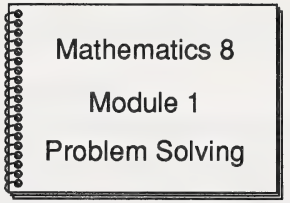
What Lies Ahead

In this section the student will learn these skills.

- making a simpler model of a complex problem in order to find a pattern
- applying the pattern to solve the problem

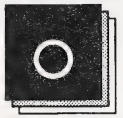
Gathering Materials

For this section the student will need these items.



(optional)

- *MATH WORKS: Problem Solving: Simplifying the Problem* (AIT)
- *SOLVE IT: Solving a Simpler Problem* (AIT)



(optional)

- *MATH STRATEGIES: Problem Solving* (SRA)

Guiding the Student

- Emphasize to the students the goal of this section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

Computer Alternative

1. Calculator Practice

The computer program you will be using has a built-in calculator to help you do computations. The following exercise will show you how to use this built-in calculator.

You will be working at the computer, so take this booklet and Disk 1 of the *MATH STRATEGIES: Problem Solving* (SRA) with you. Run the disk and wait until you see the Chapter Menu on the screen.

Type "1." Press RETURN. Follow the directions on the screen until you see the following chart.

KEY:

Plus	+
Minus	–
Times	× or *
Divided by	/
Is equal to	RETURN

Notice that you may type either [×] or [*] for multiplication. Also, since there is no [÷] key, use the [/] key for division instead. You need to use the SHIFT key when typing [+] or [*].

Type "46 + 38." Press RETURN. Here's what should be on the screen:

46 + 38 = 84

On the next line, you'll see a ":" followed by a flashing "—." This is the signal that the calculator is ready for the next calculation.

Try holding the SHIFT key and press the [=] key. As you can see, the "=" sign is not used on this calculator. Whenever you're ready for the calculator to show the answer, press RETURN instead.

Type "876." Then press the [←] key once. The 6 is erased.

When you want to erase something you have typed, use the [←] key. (Note: Once you have pressed RETURN, you cannot erase what you have typed.)

To erase all the work that is on the screen, type "E". Try it.

If you want to type a 5-digit number, say 34 672, you will have to type "34672". The calculator will not accept spaces.

2. Spy Ring

   Choose "Program 5 Using Models" from Disk 1 of the *MATH STRATEGIES: Solving Problems* (SRA) package. Then do the "Spy Ring Problem."

Note

When you see ":—" on the screen it means that you can use the built-in calculator to do calculations. After you have done the calculations, you must enter your answer and press RETURN.

Print Alternative

For Questions 3 to 5, you may act out the problems or draw diagrams in order to help you find the patterns. Apply the patterns to solve the problems.

3. There are eleven students attending a summer camp. If two students were introduced to each other at a time, how many introductions were made?

2. Computer corrected

3.

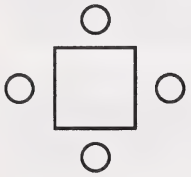
Number of Students	Number of Introductions	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4
6	15	+5

+1, +1, +1, +1

55 introductions were made.

4. What is the largest number of sections into which a circle can be divided with ten straight lines?

5. One person can be seated on each of the four sides of a square table. If 14 square tables are arranged in one long row, how many people can be seated?



4.

Number of Lines	Number of Sections	Pattern
1	2	
2	4	+2
3	6	+2
4	8	+2
5	10	+2

A circle can be divided into 20 sections with ten lines.

5.

Number of Tables	Number of Seats	Pattern
1	4	
2	6	+2
3	8	+2
4	10	+2
5	12	+2

30 people can be seated if 14 tables are arranged in one long row.

SIMPLIFYING A PROBLEM

What Lies Ahead

In this section the student will learn these skills.

- using smaller numbers to solve a problem with large numbers
- breaking problems into steps

Gathering Materials

For this section the student will need these items.

Mathematics 8
Module 1
Problem Solving

(optional)

- *SOLVE IT: Solving a Simpler Problem* (AIT)
- *MATH WORKS: Simplifying the Problem* (AIT)

(optional)

- *MATH STRATEGIES: Solving Problems* (SRA)

Guiding the Student

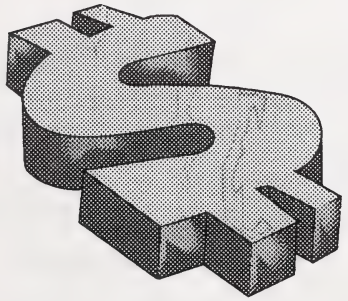
- Emphasize to the students the goal of this section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

For Questions 1 and 2, use simpler problems to help you solve the original problems. You can change the settings and the numbers, but do not change the operations.

1. The total Canadian national debt in 1977 was $29 585 900 000. In 1987 the national debt was $264 101 000 000. During this period of time, how much did Canada's national debt increase?

2. On December 31, 1986 there were 1 157 021 000 bills of Canadian paper money in circulation. Of these, 335 727 000 were one-dollar bills and 335 173 000 were twenty-dollar bills. How many bills were there of each of the other denominations (two-, five-, ten-, fifty-, hundred-, and thousand-dollar bills)?



Suggested Answers

1. In 1977 Shawn and Lisa owned 3 dogs. By 1987 they owned 20 dogs. In this period of time, how many new dogs did they get?

   $20 - 3 = 17$

   They got 17 new dogs.

   Use the same method to solve the original problem.

   $264\ 101\ 000\ 000 - 29\ 585\ 900\ 000 = 234\ 101\ 000\ 000$

   The national debt increased by $234 101 000 000.

2. On December 31, 1986 there were 800 dogs in Barrhead. Of these, 70 were collies and 80 were poodles. How many dogs were there of each of the other breeds (German shepherd, beagle, etc)?

   $$\begin{array}{r} 70 \\ +\ 80 \\ \hline 150 \end{array} \qquad \begin{array}{r} 800 \\ -\ 150 \\ \hline 650 \end{array}$$

   There were 650 dogs of other breeds in Barrhead. Use the same method to solve the original problem.

   $$\begin{array}{r} 335\ 727\ 000 \\ +\ 335\ 173\ 000 \\ \hline 670\ 900\ 000 \end{array} \qquad \begin{array}{r} 1\ 157\ 021\ 000 \\ -\ 670\ 900\ 000 \\ \hline 486\ 121\ 000 \end{array}$$

   There were 486 121 000 bills of other denominations in circulation.

Computer Alternative

3. Do "Lightning Strikes" from the *MATH STRATEGIES: Solving Problems* (SRA) package.

   You will need Disk 1. Start Disk 1. When the Chapter Menu comes on the screen, type "2" and press RETURN. You will then see the menu for Simplifying Problems.

   Type "1" for "Lightning Strikes" and press RETURN.

   Follow the instructions on the screen.

3. Computer corrected

4. Do "A Beautiful Dream" from *MATH STRATEGIES: Solving Problems* (SRA).

   Use Disk 2. From the Chapter Menu, choose "3 Problem Breakdown." Then, choose "2 A Beautiful Dream."

   Solve the problem at the computer.

4. Computer corrected

Print Alternative

In Questions 4 and 5 simplify the problems by doing them in steps.

5. The students in a Grade 8 class have a total of 51 pets. Five students have one pet each, eight students have three pets each, three students have none, and the rest have two pets each. How many students have exactly two pets?

6. Rohan charges $10 per hour to clear the neighbours' driveways. His father charges him $2 per hour for use of the snowblower. How much profit does Rohan make if he and a friend work 27 hours and they share the profits equally?

5. First find out how many pets are owned by the students who own one and three pets each.

$$\begin{aligned} 5 \text{ students} \times 1 \text{ pet each} &= 5 \text{ pets} \\ 8 \text{ students} \times 3 \text{ pets each} &= \underline{24 \text{ pets}} \\ &\quad 29 \text{ pets} \end{aligned}$$

To find out how many pets are owned by students who have two pets each, subtract the number of pets owned by students who have one and three pets each from the total number of pets.

$$51 - 29 = 22 \text{ pets}$$

To find how many students own the remaining 22 pets, divide the number of pets by 2, since each student owns 2 pets.

$$22 \div 2 = 11$$

Eleven students own two pets each.

6. Find out how much money is earned for working 27 hours.

$$\$10 \times 27 = \$270$$

Find out how much Rohan's father charges for using his snowblower for 27 hours.

$$\$2 \times 27 = \$54$$

To figure out total profit, subtract how much is owed to Rohan's father from how much money was earned in 27 hours.

$$\$270 - \$54 = \$216$$

Since Rohan and his friend split the profit, divide by 2.

$$\$216 \div 2 = \$108$$

Rohan makes $108.

WORKING BACKWARDS

What Lies Ahead

In this section the student will learn these skills.

- working backwards to solve problems
- using flow charts

Gathering Materials

For this section the students need these items.

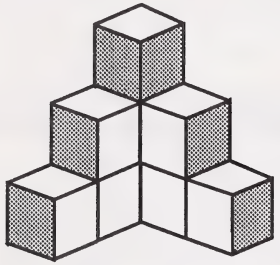
Mathematics 8
Module 1
Problem Solving

Guiding the Student

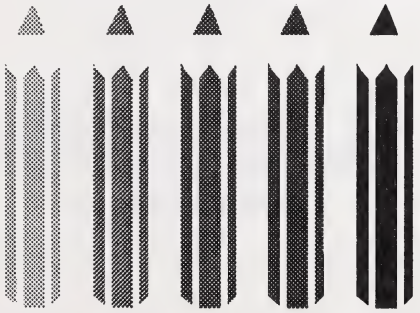
- Emphasize to the students the goal of this section.
- Help the students check their answers to the Practice Activities and correct any errors.

Practice Activities

1. Boxes can be stacked in a corner so that each layer has two less boxes than the layer below it, and the top layer has one box. How many boxes would be in the bottom layer if 36 boxes were stacked this way?

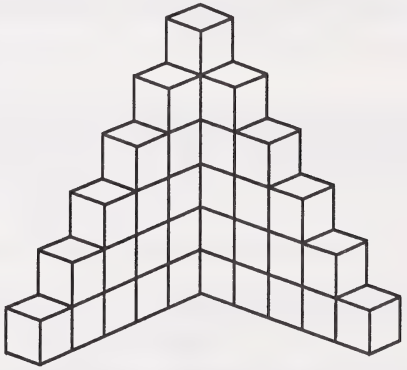


2. At the end of the school day, a teacher had 15 pencils left in her loan box. She remembered giving out 17 pencils in the morning and getting 11 back before noon, and then giving out 13 more pencils after lunch hour. How many pencils did she have at the beginning of the day?



Suggested Answers

1. Boxes are stacked from the bottom to the top. To solve the problem make a diagram beginning at the top and working down until 36 blocks have been stacked.



← 1 block
← 3 blocks
← 5 blocks
← 7 blocks
← 9 blocks
← 11 blocks

36 blocks

The top row has one block. Each consecutive lower row has two more blocks.

The bottom row has 11 blocks.

2. Working Forwards: ▒ − 17 + 11 − 13 = 15
Working Backwards: 15 + 13 − 11 + 17 = $\boxed{34}$

The teacher had 34 pencils at the beginning of the day.

3. Curly gave Larry and Moe as much money as each had. Then Larry gave Curly and Moe as much money as each had. Then Moe gave Curly and Larry as much money as each had. If each one now has $8, how much did each have at the beginning?

3. Work backwards. Use paper money and three people if necessary to act out the problem.

At the end Moe, Curly, and Larry have $8 each.

In the last step Moe gave Curly and Larry as much money as each had. In other words, he doubled their money. How much did Curly and Larry have before the last step?

Think $2 \times \square = 8$
$2 \times \boxed{4} = 8$

Curly and Larry each had $4 before the last step.

How much did Moe have before he gave away the money?

Think $\square - 2 \times 4 = 8$
$\boxed{16} - 8 = 8$

Moe had $16 before the last step.

In the middle step Larry gave Curly and Moe as much money as each had. In other words, he doubled their money. How much did Larry and Curly have before?

Think $2 \times \square = 4$ $\quad$ $2 \times \square = 16$
$2 \times \boxed{2} = 4$ $\quad$ $2 \times \boxed{8} = 16$

Curly had $2 and Moe had $8 before the middle step.

How much did Larry have before he gave away the money?
Think $\square - (2 + 8) = 4$
$\boxed{14} - 10 = 4$

Larry had $14 before the middle step.

In the first step Curly gave Larry and Moe as much as each had. In other words, he doubled their money. How much did Larry and Moe have at the beginning?

Think $2 \times \square = 14$ $2 \times \square = 8$

$2 \times \boxed{7} = 14$ $2 \times \boxed{4} = 8$

Larry had \$7 and Moe had \$4 at the beginning.

How much did Curly have at the beginning?

Think $\square - (7 + 4) = 2$

$\boxed{13} - 11 = 2$

Curly had \$13 at the beginning.

The steps can be shown in a table.

	at end	before last step	before middle step	at beginning
Moe	\$8	\$16	\$8	\$4
Larry	\$8	\$4	\$14	\$7
Curly	\$8	\$4	\$2	\$13

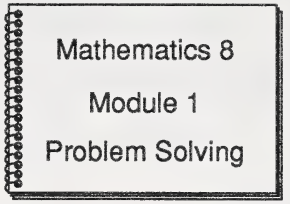
MORE THAN ONE WAY TO GO

What Lies Ahead

In this section the student will learn that there are many ways to solve one problem.

Gathering Materials

For this section the student will need these items.



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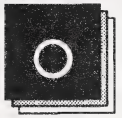
THINKABOUT: There are Many Ways to Go (AIT)



(optional)

Problem Solving Strategies (MECC)

Guiding the Student

- Emphasize to the students the goal of the section.
- Help the students check the answers to the Practice Activities and correct any errors.

Practice Activities

Computer Alternative

1. Do the programs "Diagonals" and "Squares" in the *MECC: Problem Solving Strategies* package.

Print Alternative

For Questions 2 and 3 use two different methods to solve the problems.

2. How many rectangles are in this figure?

Suggested Answers

1. Computer corrected

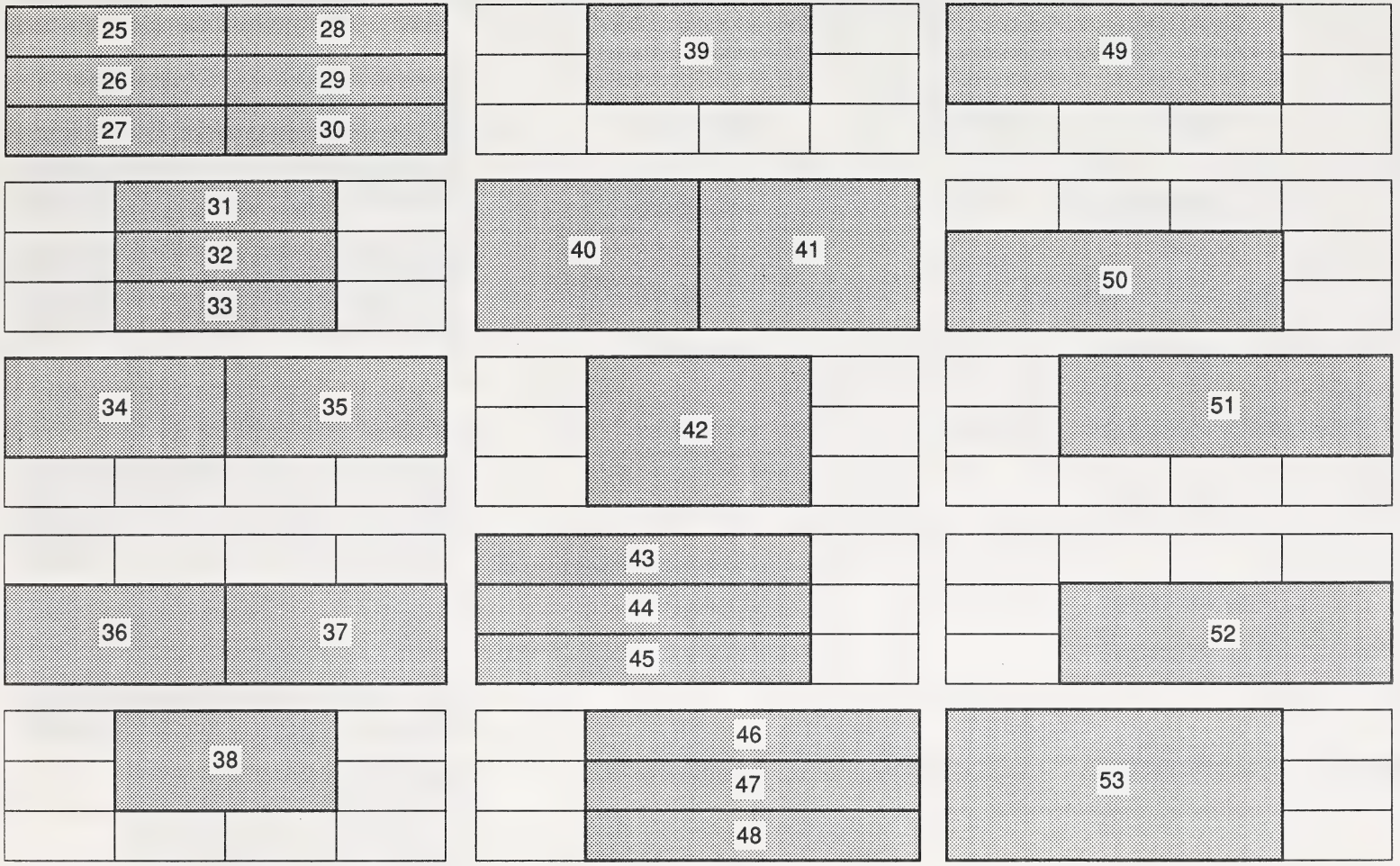
2. One method is to make an organized list of the rectangles.

1	4	7	10
2	5	8	11
3	6	9	12

13	14	15	16

17	18	19	20

21	22	23	24


25
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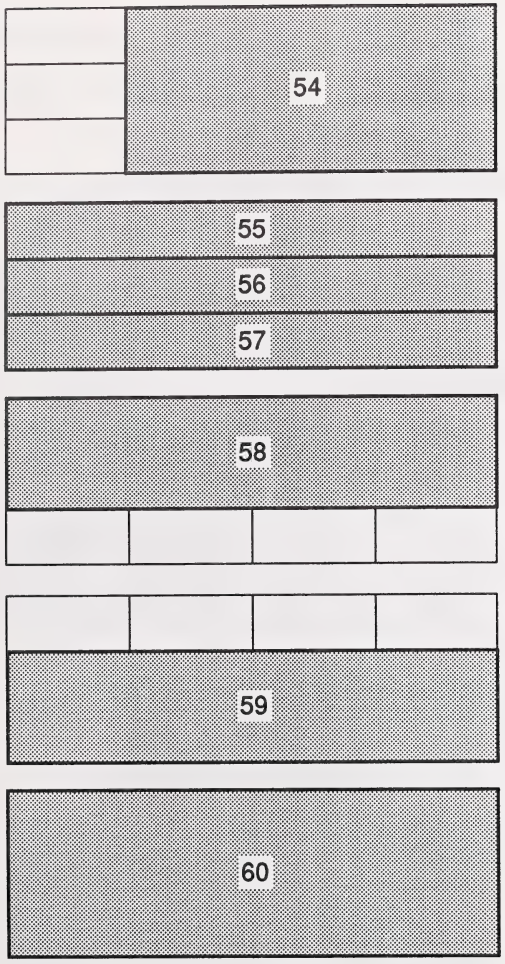
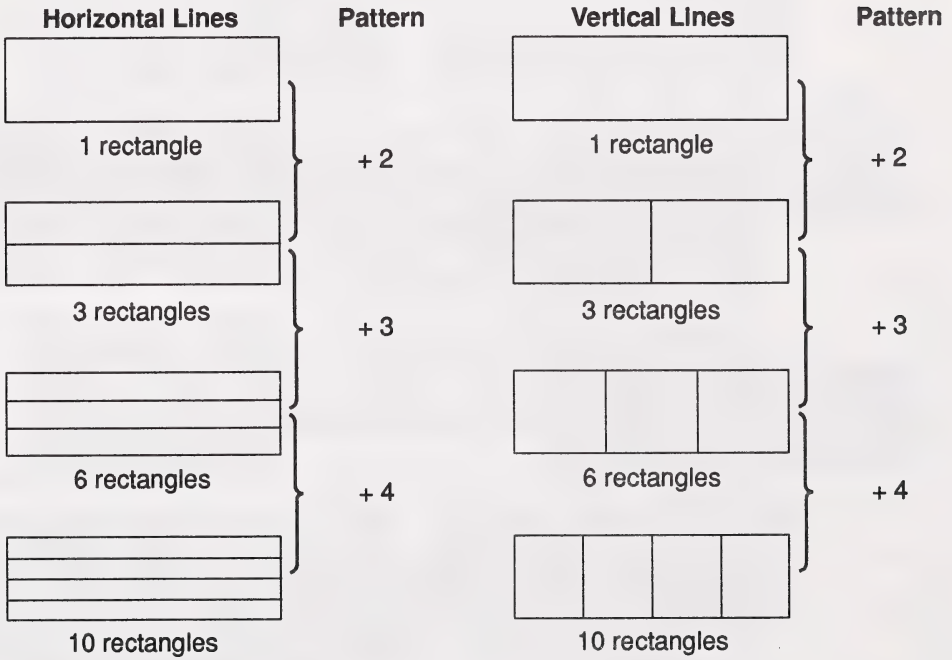
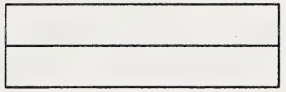
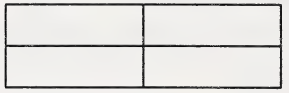
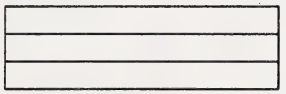
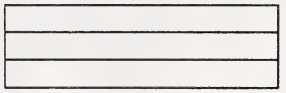
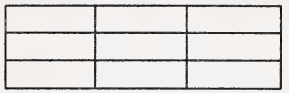
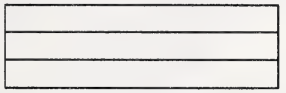


There are 60 rectangles altogether.

Another way to do 2.a. is to find a pattern. There is a pattern when lines are added to the original rectangle either horizontally or vertically.



Note there is a different pattern when lines are added in both directions.

Horizontal Lines	Vertical Lines	Lines in Both Directions	Pattern
 3 rectangles	3 rectangles	 9 rectangles	$3 \times 3 = 9$
 6 rectangles	3 rectangles	18 rectangles	$6 \times 3 = 18$
 6 rectangles	6 rectangles	 36 rectangles	$6 \times 6 = 36$
 6 rectangles	10 rectangles	60 rectangles	$6 \times 10 = 60$

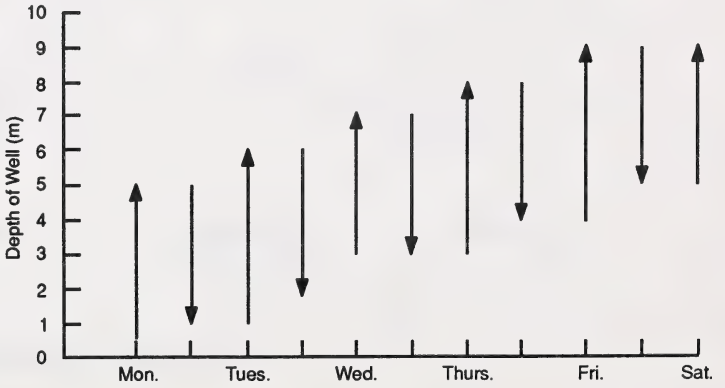
3. A well is 10 m deep. A salamander climbs up 5 m during the day and climbs down 4 m at night. If the salamander started at the bottom on Monday, on what day will it get to the top?

3. One way to solve this problem is to make a table.

	Day	Night
Monday	5 m	5 – 4 = 1 m
Tuesday	5 + 1 = 6 m	6 – 4 = 2 m
Wednesday	5 + 2 = 7 m	7 – 4 = 3 m
Thursday	5 + 3 = 8 m	8 – 4 = 4 m
Friday	5 + 4 = 9 m	9 – 4 = 5 m
Saturday	5 + 5 = 10 m reaches the top	

The salamander will reach the top on Saturday.

Another way to solve the problem is to make a diagram.



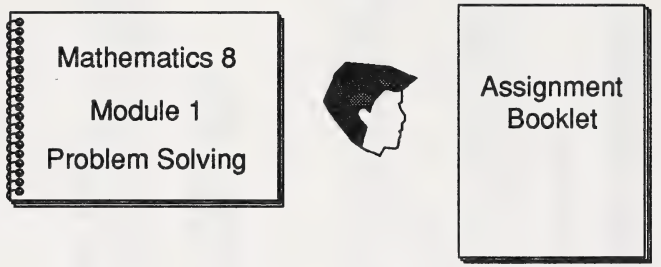
MODULE CONCLUSION

What Lies Ahead

The student is now ready to do the assignment in the assignment booklet. The student will be graded on the work done in this booklet.

Gathering Materials

For this section the student will need these items.



Guiding the Student

- Students should complete the assignment independently.
- Provide each student with a grade and feedback to the assignment.
- Be sure to look for evidence of understanding the problem, developing and trying the plan, and looking back. Answers should be expressed in complete sentences, and the units should be included.

Suggested Answers to Assignment Booklet

Part 1: Multiple-Choice Questions

30

Each of the following questions has four suggested answers, one of which is better than the others. Indicate your choice by writing the letter in the blank on the response page at the right.

1. Alec is two years younger than Ben, and Ben is two years younger than Chris. Which question can you answer using this information?

   A. How old is Alec?
   B. How old is Chris?
   C. How much younger is Ben than Alec?
   D. How much younger is Alec than Chris?

2. One hamburger patty has a mass of about 120 g. How much hamburger meat is needed to serve double burgers to 25 students? (Be sure to choose the most reasonable answer.)

   A. 60 g
   B. 600 g
   C. 6000 g
   D. 60 000 g

3. Which strategy is **not** one of the techniques used to identify the necessary parts of a problem?

   A. eliminating unnecessary information
   B. restating the problem
   C. finding a pattern
   D. changing the setting of the problem

4. A city lot in the shape of a rectangle has a value of $20 000. What is the value of a similar shaped lot that is twice as wide as the first lot?

   A. $80 000
   B. $100 000
   C. $60 000
   D. $40 000

Part 1 Response Page

1. D
2. C
3. C
4. D

Part 1 (continued)

5. Consider the following problem:

   Three-eyed skonks and four-eyed zunds live on the planet Curios. A group of these creatures together have 34 eyes. How many of this group are skonks?

   Which one of the following strategies would you use to solve this problem?

   A. making a sketch
   B. finding a pattern
   C. simplifying the problem
   D. making a table

6. In which stage of the four-stage process for problem solving should you make an estimate?

   A. developing a plan
   B. looking back
   C. trying the plan
   D. understanding the problem

7. In which stage of the four-stage process for problem solving should you perform the calculations that may be needed to solve the problem?

   A. developing a plan
   B. looking back
   C. trying the plan
   D. understanding the problem

8. Consider the following problem:

   Brian and Cory each have free ride tickets for the Winter Carnival. If Brian gives Cory two tickets they will have the same number. But if Cory gives Brian two tickets, Brian will have twice as many. How many tickets does each boy have?

   Which strategy should you use to solve this problem?

   A. guessing-checking-revising
   B. finding a pattern
   C. simplifying the problem
   D. drawing a sketch

Part 1 Response Page (continued)

5. D

6. D

7. C

8. A

Part 1 (continued)

9. Consider the following problem:

   Justin and Jenny want to buy their mother a camera for a birthday gift. (1)

   They picked out a camera with a flash unit and optional carrying case. (2)

   Her birthday is in ten days, so they want to buy the gift right away. (3)

   The camera set, which normally costs $45.00, is on sale for $29.00. (4)

   This sale price is only available at one store, and the sale ends in three days. (5)

   Justin has $12.57 in his savings, and Jenny has $15.91 in hers. (6)

   Do the children have enough money to buy this gift? How much more do the children need?

   Each statement for the problem is numbered clearly. Which of these statements are unnecessary in order to answer the problem?

   A. Statements 1, 2, 3, and 4 are unnecessary.
   B. Statements 3, 4, 5, and 6 are unnecessary.
   C. Statements 2, 3, 5, and 6 are unnecessary.
   D. Statements 1, 2, 3, and 5 are unnecessary.

10. What is the answer to the problem in Question 9?

    A. $16.52
    B. $0.52
    C. $45.52
    D. $16.00

11. Consider this problem:

    The houses on Granada Court are numbered from 1 to 153 with no numbers missing. How many of these house numbers contain the digit 3 at least once?

    What is the best strategy to solve this problem?

    A. making a sketch
    B. making an organized list
    C. simplifying the problem
    D. guessing-checking-revising

Part 1 Response Page (continued)

9. D

10. B

11. C

Part 1 (continued)

12. Consider the following problem:

    A table can seat 6 with two people at each long side and one at each end. How many tables are needed to seat 26 people if they are placed in one long row?

    Which strategy is needed to solve this problem?

    A. making a list
    B. finding a pattern
    C. working backwards
    D. simplifying the problem

13. There are 16 pens and pencils in a box. There are 6 more pens than pencils. To find the number of pencils, subtract 6 from 16, then divide the result by 2. Next, add 6 to this result to get the number of pens. Which problem below can be solved using exactly the same operations as this problem?

    A. Donnie earns $8 per week more than Lance. Together they earn $88 per week. How much less does Lance earn than Donnie?
    B. Douglas is 10 cm taller than Allan. Their total combined height is 354 cm. What is the height of each boy?
    C. Together Buster and Ernie have 29 baseball caps. If Buster has 7 more than Ernie, how many baseball caps does Ernie have?
    D. A container of orange juice costs $0.40 more than a same-size container of apple juice. Both together cost $2.18. What is the cost of the orange juice?

14. Consider this problem:

    A, B, and C are waiting in a line-up to buy tickets. In how many different ways can they stand in line?

    Which of the organized lists below would provide the answer to the above problem?

    A. A-A-B A-B-A B-B-A B-B-C C-C-A C-C-B
    B. A-B-C B-C-A C-A-B
    C. A-B-C-D B-C-A-D C-A-B-D D-A-B-C
    D. A-B-C A-C-B B-A-C B-C-A C-A-B C-B-A

Part 1 Response Page (continued)

12. C

13. C

14. D

Part 1 (continued)

15. Girard normally assembles an average of 142 items each week at his factory job. On the first four days of this week he has assembled 24, 30, 33, and 28 items. How many items must he assemble on Friday to maintain his average?

    A. 27 items
    B. 28 items
    C. 29 items
    D. 30 items

Part 1 Response Page (continued)

15. A

Total for Part 1 = _____ (Maximum possible: 30 marks)

Part 2: Short-Answer Questions

40

Answer the following questions on the response page at the right. Give complete answers and show all necessary work.

4 1. List the four stages that can be used to solve any problem.

3 2. You have a package of 500 sheets of typing paper. In your own words explain how to find the thickness of one sheet of paper.

4 3. Jonty has a large number of 25-cent coins, 10-cent coins, and 5-cent coins. Make an organized list showing all the possible coin combinations which he could use to pay the debt of 50 cents that he owes to his friend Benjamin.

Part 2 Response Page

1. The four stages are understanding the problem, developing a plan, trying the plan, and looking back.

2. Answers will vary. Here is an example:

   Measure the thickness of 500 sheets in centimetres. Then divide the thickness by 500. This will give you the thickness of one sheet.

3. Jonty can pay the debt with these coins:

   - two 25-cent coins
   - one 25-cent coin, two 10-cent coins, and one 5-cent coin
   - five 10-cent coins
   - four 10-cent coins and two 5-cent coins
   - three 10-cent coins and four 5-cent coins
   - two 10-cent coins and six 5-cent coins
   - one 10-cent coin and eight 5-cent coins
   - ten 5-cent coins

Part 2 (continued)

5

4. You have learned several strategies that can be used for solving problems. Some of these are listed below.

- using objects
- drawing a sketch
- making a table
- simplifying the problem
- making an organized list
- finding a pattern
- working backwards
- guessing-checking-revising

For each of the following problems, select and write the name of the strategy that will be the most useful for solving that problem.

a. The distance from Winnipeg to St. John's in the east is 3162 km. From Winnipeg to Vancouver in the west is 1849 km. What is the distance from St. John's to Vancouver?

b. Monic spent one-half of her money plus $5 at the first shop. At the second shop she spent one-half of her money plus $5. If Monic was left with $5, how much did she spend at the first shop?

c. Frank earns 25 cents each time he dries the dishes and 15 cents each time he sweeps the floor. After a week he has done 12 jobs and earned $2.50. How many times did he sweep the floor?

d. Thirty people came to a party and were introduced. How many handshakes were exchanged if two people shook hands each time?

e. Ruth earned $18 more than Axel. Axel earned $13 more than Sissy who, in turn, earned $8 more than Wayne. If their total earnings were $178, how much did Wayne earn?

f. How many triangles are there in this figure?

Part 2 Response Page (continued)

4. a. simplifying the problem

   b. working backwards

   c. making a table

   d. finding a pattern

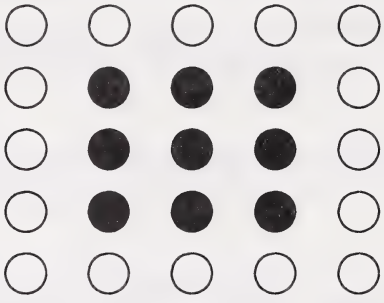
   e. guessing-checking-revising

   f. making an organized list

Part 2 (continued)

5. For each of the following problems, select and write the names of two strategies that can be used for solving that problem.

2 a. Sixteen white chips and nine black chips are used to make the design shown. If you make a similar design with eight chips on each side, how many white chips and black chips would you need?



2 b. Roger has a bucket that holds exactly three litres and another bucket that holds exactly five litres. How can Roger measure exactly one litre by using only these two buckets?

2 c. 26 all-terrain vehicles are on display. Some of them have 3 wheels and some have 4 wheels. A child counted 94 wheels in all. How many 3-wheeled all-terrain vehicles are in the display?

3 6. Make a sketch that can be used to solve the following problem:

Peter lives 155 blocks from work. One morning he drove 48 blocks before realizing that he had forgotten his briefcase. He returned home for his briefcase and then went to work. How far did he drive to get to work that morning?

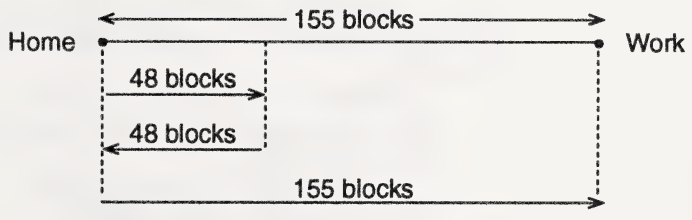
Part 2 Response Page (continued)

5. a. finding a pattern and guessing-checking-revising

   b. finding a pattern and drawing a sketch

   c. making a table and guessing-checking-revising

6.



Part 2 (continued)

6

7. Consider the following problem:

   Mr. Jamieson has $2309.34 in his bank account. He wants to buy a birthday present for his 13-year old step-son. After shopping for a few days, he decides to get him a skateboard valued at $89.95. This is a sale price and appears to be a good value. He buys the gift by writing a cheque. What is the new balance in his bank account after making this purchase?

   a. Underline the essential information in the problem.

   b. Using the underlined parts, restate the problem in your own words.

6

8. Consider the following problem:

   One weekend Linda spent three times as much on clothes as she spent on recreation. If she spent $8.00 on make-up, how much did she spend on clothes?

   a. There is not enough information to solve this problem. Supply the missing information.

   b. Which information is not needed?

3

9. Consider the following problem:

   At its closest point, the planet Jupiter is 628 800 000 km from Earth. The planet Saturn is 598 600 000 km beyond Jupiter. At its closest point, how far is Saturn from Earth?

   Change the setting of the problem to something familiar to you. You can do this by changing numbers and details – but do NOT change the required mathematical operations.

Part 2 Response Page (continued)

7. a. Mr. Jamieson has $2309.34 in his bank account. He wants to buy a birthday present for his 13-year old step-son. After shopping for a few days, he decides to get him a skateboard valued at $89.95. This is a sale price and appears to be a good value. He buys the gift by writing a cheque. What is the new balance in his bank account after making this purchase?

   b. Mr. Jamieson has $2309.34 in his bank account. He writes a cheque for $89.95. What is the new balance in his account after making this purchase?

8. a. Answers will vary. Here is an example:

   Linda spends $10.00 on recreation.

   b. She spent $8.00 on make-up.

9. Answers will vary. Here is an example:

   Jimmy lives 5 km east of the arena. Louis lives 3 km farther east than Jimmy. How far east of the arena does Louis live?

Total for Part 2 = _____ (Maximum possible: 40 marks)

Part 3: Problems

30

Choose any **five** of the following six problems and solve them. Be sure to show all of your work on the response page at the right.

6

1. In each round of a tennis tournament, the players are grouped into pairs for a match with only the winner advancing to the next round. If 32 players start in the tournament, how many matches must be played before a single winner can be declared?

6

2. A drawer contains two white socks and four black socks. How many different ways are there of removing the socks one at a time?

6

3. A certain computer "beeps" whenever the 4-key or the 8-key is typed. If Janeen types all the numbers from 34 to 84, how many times will the computer "beep"?

Part 3 Response Page

1. 31 matches must be played.

2. There are 15 ways.

w - w - b - b - b - b
w - b - w - b - b - b
w - b - b - w - b - b
w - b - b - b - w - b
w - b - b - b - b - w
b - w - b - b - b - w
b - w - b - b - w - b
b - w - b - w - b - b
b - w - w - b - b - b
b - b - w - b - b - w
b - b - w - w - b - b
b - b - w - b - w - b
b - b - b - w - b - w
b - b - b - w - w - b
b - b - b - b - w - w

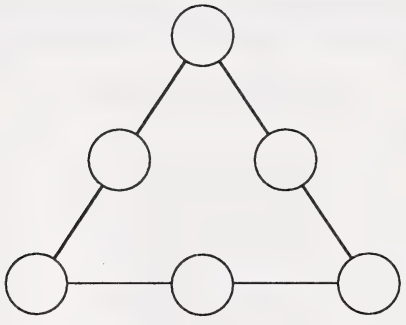
3. The computer will beep 25 times.

Part 3 (continued)

6 4. A rectangular pan of squares is 30 cm by 46 cm. When four equal rows are cut from one end, the remaining figure is a square. How wide is each row that is cut?

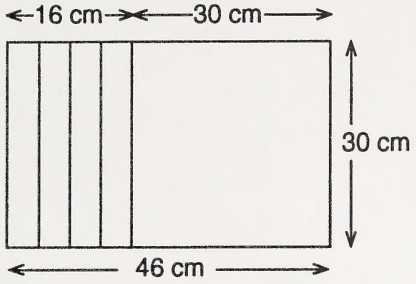
6 5. In a bag of jelly beans, 40% are red, 30% are yellow, 20% are black, and the remaining six jelly beans are green. How many jelly beans does the bag contain?

6 6. A magic triangle is one in which the numbers in the circles on each side of the triangle add up to the same number. You may use each number only once.



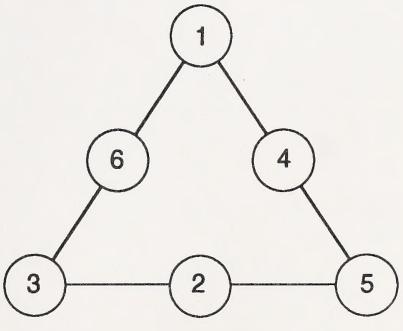
Use the numbers 1, 2, 3, 4, 5, and 6 so that each side of the triangle adds up to 10.

Part 3 Response Page (continued)

4. 

Each row that is cut is 4 cm wide.

5. There are 60 jelly beans in the bag.

6. 

Total for Part 3 = _____ (Maximum possible: 30 marks)

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